

# Principal's Research Review

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## Building a Foundation for Success in Secondary School Mathematics

By Eric Knuth and Amy Ellis

**T**he mathematics education of K–12 students has been a topic of national concern since the publication of *A Nation at Risk: The Imperative for Educational Reform* (National Commission of Excellence in Education, 1983). In 2006, when President Bush established the National Mathematics Advisory Panel to recommend ways “to foster greater knowledge of and improved performance in mathematics among American students” (Presidential Executive Order 13398), it once again put a focus on the need for U.S. students to acquire greater knowledge of and improve performance in mathematics. Both national and international assessments show a decline in mathematics achievement in the U.S. beginning in late middle school—a point that marks a significant mathematical transition from the concrete, arithmetic reasoning of elementary school mathematics to the increasingly complex, abstract algebraic reasoning required for high school mathematics and beyond.

One aspect of the educational system that influences student learning in mathematics is the content and sequencing of middle level and high school mathematics curricular topics to better prepare students for entry into and success in algebra and beyond. The reasons to focus on algebra are twofold. First, although

students study important topics within many areas of secondary school mathematics, algebra is often viewed as the linchpin to students’ success in mathematics given its foundational role in *all* areas of mathematics (National Mathematics Advisory Panel, 2008; RAND Mathematics Study Panel, 2003). Algebra also provides the mathematical tools that are used to represent and analyze quantitative relationships, to model situations, to solve problems, and to state and prove generalizations (National Council of Teachers of Mathematics [NCTM], 2000; RAND Mathematics Study Panel). Second, algebra is widely recognized as the gatekeeper to future educational and employment opportunities (Ladson-Billings, 1998; Moses & Cobb, 2001; National Research Council [NRC], 1998). As Schoenfeld (1995) aptly stated:

Algebra has become an academic passport for passage into virtually every avenue of the job market and every street of schooling. With too few exceptions, students who do not study algebra are therefore relegated to menial jobs and are unable often to even undertake training programs for jobs in which they might be interested. They are sorted out of the

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opportunities to become productive citizens in our society. (pp. 11–12)

## A Balanced and Coherent Approach to Mathematics

Student success in mathematics often means different things to different people, ranging from being fluent with computational procedures to being able to solve complex problems. As outlined in the NRC's *Adding It Up*, mathematical proficiency consists of five separate, yet intertwined strands:

- Conceptual understanding—comprehension of mathematical concepts, operations, and relations
- Procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- Strategic competence—ability to formulate, represent, and solve mathematical problems
- Adaptive reasoning—capacity for logical thought, reflection, explanation, and justification
- Productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in the value of diligence and in one's own efficacy. (Kilpatrick, Swafford, & Findell, 2001, p. 116)

These strands are interwoven and interdependent in the development of mathematical proficiency. Thus, a balanced and coherent curricular approach provides students with coordinated, long-term opportunities to develop all strands of proficiency.

### Critical Foundations of Algebra and Algebraic Reasoning

Most definitions view algebra as the generalization and formalization of patterns and constraints and of algebraic reasoning as both generalized arithmetic reasoning and generalized quantitative reasoning (Kaput, 1999). Even before the middle level, teachers can focus on algebraic reasoning to develop students' abilities to generate, represent, and justify generalizations about the properties of arithmetic. Throughout the 6–12 mathematics curriculum, there are four major content domains that play critical roles in the development of algebra and algebraic

reasoning: numbers and operations; proportional reasoning; algebraic symbols and variables; and patterns, relations, and functions.

## The Four Content Domains of the 6–12 Mathematics Curriculum

- Numbers and Operations
- Proportional Reasoning
- Algebraic Symbols and Variables
- Patterns, Relations, and Functions

### Numbers and Operations

Numbers and operations includes an understanding of different types of numbers, their properties, and how to operate on them. In preparation for the middle level, students should have a solid understanding of whole numbers, including proficiency with whole number operations, such as addition, subtraction, multiplication, and division; understanding decimal notation as an extension of place value; reading, representing, and interpreting numbers with verbal descriptions, geometric models, and mathematical models; making sense of large and small numbers with scientific notation; and modeling and solving problems involving number-theory concepts such as prime and composite numbers, divisibility and remainders, greatest common factors, and least common multiples (Kilpatrick et al., 2001). Research shows that instruction that emphasizes an understanding of algorithms before using them leads to an increase in both conceptual and procedural knowledge (Fuson et al., 1997; Rittle-Johnson & Alibali, 1999; Siegler, 2003), so there is evidence that teachers should promote understanding over rote practice with algorithms in order to help students develop procedural fluency.

At the middle level, students should also develop a solid understanding of integers and properties of numbers, including concepts of negative numbers;

operations with integers; and an understanding of general properties of numbers, including the additive and multiplicative properties of equations and inequalities, the commutativity and associativity properties of addition and multiplication, the distributive property, inverses and identities for addition and multiplication, and the transitive property (Conference Board of the Mathematical Sciences, 2001; Kilpatrick et al., 2001). Research shows that students at the secondary and even college levels can have difficulty working with integers (Bruno, Espinel, & Martinon, 1997). Students can develop a stronger understanding of operations with integers by using metaphors for modeling operations with negative numbers, which can include examples such as elevators, thermometers, debts and assets, hot air balloons, or arrows on a number line (Carson & Day, 1995; Moreno & Mayer, 1999).

In high school, students should develop a solid understanding of real numbers, understanding the relationships among whole, integral, rational, and irrational numbers. A high school curriculum should build on the ideas developed in middle school, emphasizing an understanding of the number line as a representation of the real numbers; complex operations on real numbers, including raising to a power, extracting roots, taking opposites and reciprocals, and determining absolute value; and the ability to select appropriate operations in complex problem-solving situations. High school students should also develop an understanding of number theory, moving from a focus on the properties of numbers to a focus on the properties of the natural, integer, rational, real, and complex number systems. Teachers should emphasize connections between number theory and algebraic structures and develop problems relying on complex counting procedures such as the union and intersections of sets.

### **Proportional Reasoning**

Proportional reasoning is a significant milestone in mathematical development. The ability to reason proportionally develops slowly over time and should be highlighted at all levels as students begin to reason algebraically. According to the NCTM (1989),

proportional reasoning is “of such great importance that it merits whatever time and effort must be expended to assure its careful development” (p. 82). In preparation for proportional reasoning at the middle level, students should have a solid understanding of rational numbers, which is a number that can be written as a ratio of two integers in the form  $a/b$  (where  $b$  is not zero). Students should be able to understand fractions and their different meanings: fractions can represent part to whole relationships, ratios, division, or single entities on a number line. Students should enter the middle grades fluent in operating with fractions and decimals and should be able to easily convert between fractions, decimals, and percents. Teachers should encourage a solid qualitative understanding of fractions: students should be able to estimate calculations, compare relative sizes of fractions, and express order relationships among fractions using the appropriate symbols ( $>$ ,  $<$ ,  $\neq$ ).

In middle school, teachers should focus on ratios and rates, which can serve as a foundation for building future algebraic concepts such as linear functions, patterns, and graphs. Proportional reasoning problems can help students create ratios because they embody a multiplicative relationship between the quantities in the situation. For instance, teachers can present a scenario in which a snail moves 10 cm in 4 seconds. By asking students to express the snail’s speed in as many different ways as they can think of, teachers can help students develop many different ratios (5 cm in 2 seconds, 20 cm in 8 seconds, or 15 cm in 6 seconds). This can also encourage the development of unit rates in which students realize that the snail moves 2.5 cm per second. Problems like these can help students connect ratios and rates to multiplication and division, ultimately developing many equivalent ratios (Lobato & Thanheiser, 2002).

Proportional reasoning has been described as the gateway to higher mathematics, including algebra, geometry, probability, and statistics. However, U.S. middle level students have not performed well on even simple proportion problems. On the 1996 National Assessment of Educational Progress exam, only 12% of eighth-grade students correctly solved

a basic proportion problem comparing two speed-related rates (Wearne & Kouba, 2000). Middle level mathematics should emphasize ratios and proportions, applying proportional reasoning to a variety of problems including percents, scale drawings, speed, similarity of geometric figures, and other phenomena. Research has shown that students with real-world curricular experiences significantly outperform students in classes with traditional instructional methods (Ben-Chaim et al., 1998).

Students in algebra classes at the middle and high school levels should develop a solid understanding of linear functions, because it represents the first experience students have with functions and relations. Specifically, algebra courses should emphasize the connections between linear functions and proportions. Any direct proportion situation can be written as  $y = mx$ , which can be graphed as a line. Teachers should therefore introduce linear functions as a way to represent a constant rate of change, and students should be able to graph functions; understand the meaning of a constant slope; solve linear equations and inequalities; and translate between tables of data, linear equations, and graphs (Lobato & Ellis, in press). In high school, students should be able to solve systems of linear equations and graph linear inequalities. Linear functions provide an ideal way for algebra teachers to model real-world phenomena because constant rates of change occur in many different situations. For example, examine the relationship between the number of times a cricket chirps and the temperature outside: the rate of chirps increases linearly as the temperature increases. Research suggests that students who develop an understanding of linear functions through modeling constant rates of change will develop a more solid understanding of slope and function relationships (Ellis, 2007).

### **Algebraic Symbols and Variables**

One of the major characteristics of making the transition from arithmetic to algebra is the appropriation of literal symbols to represent unknowns, parameters, or varying quantities (Kieran, 1992). Elementary school mathematics is heavily answer oriented

and does not typically focus on the representation of relations. As students enter the middle grades, the mathematics curriculum should focus on writing and operating on symbolic equations and expressions. Students should learn how to translate English into symbolic expressions (e.g., expressing “five more than a number” as “ $5 + x$ ”), and should master rule-based activities such as collecting like terms, factoring, expanding, substituting, solving equations, and simplifying expressions. Middle level mathematics curricula must provide students with many opportunities to work with algebraic expressions in a variety of ways, including generating equivalent expressions, adding and subtracting expressions, evaluating with numerical substitution, and working with exponents and roots. Instruction should include the introduction of the properties of simplifying algebraic expressions, such as the commutative, associative, and distributive properties.

Although symbolic manipulation skills are critical, teachers should emphasize a conceptual understanding of the symbols and rules they introduce. Overly rule-based instruction does not give students opportunities to create meaning for the rules which can lead to forgetting, unsystematic errors, reliance on visual clues, and poor strategic decisions (Booth, 1984; Kirshner & Awtry, 2004; Wenger, 1987). Instead, teachers should introduce content that focuses on big ideas (Prawat, 1991) and gear instruction so that it connects with students’ experiences, knowledge, and strengths (Smith, diSessa, & Roschelle, 1993). For example, teachers can rely on students’ understanding of speed and connect to their experiences with running races in order to develop the notions of ratio and rate.

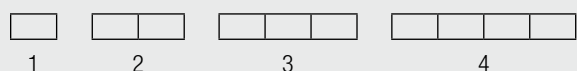
In high school, students will learn to make meaningful use of symbols. Students will build on what they learned in the middle grades to help them connect algebra with geometry; link expressions and functions; and understand the role played by literal symbols, such as unknowns, variables, and parameters. Students’ facility with manipulating symbolic expressions will grow as they practice simplifying complex expressions involving exponents and fractional exponents, radicals, and rational expressions.

These skills will enable students to focus on the meaning of complex functions, find roots of polynomial functions, and understand the fundamental theorem of algebra.

### Patterns, Relations, and Functions

A major element of algebraic reasoning is the emphasis on patterns and the ability to generalize those patterns algebraically. Students should be exposed to mathematical patterns at every grade level, with greater prominence as students enter the middle grades. Students should leave elementary school with the ability to detect and generate patterns in number, table, or pictorial representations. In middle school, students should focus on reasoning with basic functions in equation, table, and graph form. Teachers should emphasize patterns that are associated with linear, quadratic, and exponential functions, encouraging the development of students' generalization skills by helping them create algebraic rules of those patterns. For example, teachers might ask students to find an algebraic rule that describes the number of line segments needed to build the  $n$ th shape in the geometric pattern below (Figure 1 displays the first four shapes in the pattern); in this case, an algebraic rule is  $3n + 1$ .

Figure 1.



Pattern-eliciting tasks can promote the use of number, number sentences, and functions as objects for reasoning algebraically (Blanton & Kaput, 2002). However researchers have shown that students can recognize multiple patterns in any one problem but struggle to identify which patterns are mathematically viable (English & Warren, 1995; Stacey & MacGregor, 1997). Smith (2002) advocates shifting instruction away from describing static patterns toward thinking about how a pattern can be extended, i.e., describing change. By focusing on change, students will choose a unit, engage in mathematical generalization, and ultimately describe a pattern algebraically.

High school curricula should move beyond pattern identification and generalization to emphasize analyzing properties among families of functions. Students should be able to understand relationships between families of polynomial (linear, quadratic, and higher-order) functions, as well as graph and solve their equations. Once students have developed a strong understanding of families of functions, they can explore nonpolynomial functions such as exponential, square root, logarithmic, absolute value, and rational functions. By building on their understanding of patterns and relations, students will be able to develop generalizable techniques for graphing, manipulating, and solving equations, transforming functions to match data, and analyzing the effect of parameters (Smith, 2002).

In focusing on the critical foundations for success in algebra, we do not advocate a particular curricular approach (e.g., reform-based, traditional, discovery) or emphasis (e.g., conceptual understanding, computational fluency, problem solving). Stakeholders can debate the various curricular approaches and emphases, yet what is most important is a balanced and coherent curricular approach to help students engage in and develop the five strands of mathematical proficiency. A curriculum should allow students to develop their algebraic reasoning skills in a coordinated and coherent manner over the course of their secondary schooling, understand algebraic relationships and connect those ideas to other content domains.

### Laying the Foundation for Success: What Can Principals Do?

Principals can play a significant role in the effort to improve the teaching and learning of mathematics through well-designed teacher professional development and articulation of curricular content.

#### Teacher Professional Development

Arguably, one of the most important influences on teachers' instructional practices and, ultimately, on student learning is a teacher's knowledge of mathematics (Borko & Putnam, 1996; Hill, Rowan, & Ball, 2005). This is especially important at the middle

level where many teachers were trained in elementary teacher education programs and may lack the depth of mathematical knowledge that their high school counterparts possess. This is troubling because middle level mathematics curricula are much more demanding than elementary school mathematics curricula, and the central mathematical ideas of middle school are as difficult conceptually as any ideas in the K–12 mathematics curriculum (NRC, 2000).

The (in)adequacy of middle level teacher preparation is a significant problem that cannot be solved without a substantial investment in mathematics content-based professional development. Beyond more mathematics courses, teachers need opportunities to acquire the mathematics knowledge needed for teaching middle school, that is, knowledge that is tailored to the work of teaching middle level mathematics (see CBMS, 2001, for a detailed description).

In the case of hiring decisions, for example, priority might be placed on hiring middle level mathematics teachers who have advanced preparation in mathematics. In the case of staffing decisions, reassigning a middle level mathematics teacher who has had extensive mathematics-based professional development to social studies instruction is not an optimal use of district resources, even if it solves a school-level staffing challenge.

### Curriculum Focus and Articulation

In preparing students for success in algebra and beyond, the content domains require a focused and coherent curricular progression—one that is well-articulated across the grades. A well-articulated curriculum “gives teachers guidance regarding important ideas or major themes which receive special attention at different points in time. It also gives guidance about the depth of study warranted at particular times and when closure is expected for particular skills or concepts” (NCTM, 2000, p. 16).

Principals can play a critical role in ensuring that their students receive a focused and coherent curricular progression by striving to achieve K–12 articulation. As a starting point, *Curriculum Focal Points* (NCTM, 2006) offers K–8 curricular direction to teachers and principals by identifying impor-

tant mathematical topics at particular grade levels, topics that form the foundation for algebra and more advanced mathematics (the four content domains are also well represented and discussed in more detail).

Principals can also gather teachers from different grade levels to discuss the curricular progression of topics—not only for achieving curricular coherence and articulation but also for helping teachers develop algebraic ideas and to connect them to the rest of the mathematics that students encounter. Moreover, such cross-grade level discussions can support elementary school teachers in their efforts to develop basic ideas of algebra (as generalized arithmetic) in the activities they present to their students (Kilpatrick, Swafford, & Findell, 2001).

There is no easy fix to improving the mathematics knowledge and performance of secondary school students. A balanced and coherent curricular approach, one that focuses on the four content domains—numbers and operations; proportional reasoning; algebraic symbols and variables; and patterns, relations, and functions—is needed to prepare students for success in algebra. Principals can play a key role in improving the mathematics knowledge and performance of secondary school students by ensuring that their teachers have the mathematical preparation necessary to effectively teach mathematics and that students receive a focused, coherent, and well-articulated curriculum. [PRR](#)

### References

- Ben-Chaim, D., Fey, J. T., Fitzgerald, W. M., Benedetto, C., & Miller, J. (1998). Proportional reasoning among 7th-grade students with different curricular experiences. *Educational Studies in Mathematics*, 36, 247–273.
- Blanton, M., & Kaput, J. (2002, April). *Developing elementary teachers' algebra “eyes and ears”*: Understanding characteristics of professional development that promote generative and self-sustaining change in teacher practice. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.
- Booth, L. R. (1984). *Algebra: Children's strategies and errors*. Windsor, UK: NFER-Nelson.
- Borke, H., & Putnam, R. (1996). Learning to teach. In R. Calfee & D. Berliner (Eds.), *Handbook of educational psychology* (pp. 673–725). New York: Macmillan.
- Bruno, A., Espinel, M. C., & Martinon, A. (1997). Prospective teachers solve additive problems with negative numbers. *Focus on Learning Problems in Mathematics*, 19(4), 36–55.

- Carson, C. L., & Day, J. (1995). *Annual report on promising practices: How the algebra project eliminates the “game of signs” with negative numbers*. San Francisco: Far West Lab for Educational Research and Development. (ERIC Document Reproduction Service No. ED 394 828).
- Conference Board of the Mathematical Sciences. (2001). *The Mathematical Education of Teachers*. Providence RI and Washington DC: American Mathematical Society and Mathematical Association of America.
- Ellis, A. B. (2007). Connections between generalizing and justifying: Students’ reasoning with linear relationships. *Journal for Research in Mathematics Education*, 38(3), 194–229.
- English, L., & Warren, E. (1995). General reasoning processes and elementary algebraic understanding: Implications for initial instruction. *Focus on Learning Problems in Mathematics*, 17(4), 1–19.
- Fuson, K. C., Wearne, D., Hiebert, J. C., Murray, H. G., Human, P. G., Olivier, A. I., Carpenter, T. P., & Fennema, E. (1997). Children’s conceptual structures for multidigit numbers and methods of multidigit addition and subtraction. *Journal for Research in Mathematics Education*, 28(2), 130–162.
- Hill, H., Rowan, B., & Ball, D. (2005). Effects of teachers’ mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2), 371–406.
- Kaput, J. (1999). *Teaching and learning a new algebra with understanding*. Unpublished manuscript. University of Wisconsin-Madison: National Center for Improving Student Learning & Achievement in Mathematics and Science.
- Kieran, C. (1992). The learning and teaching of school algebra. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 390–419). New York: Macmillan Publishing Company.
- Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, D.C.: National Academies Press.
- Kirshner, D., & Awtrey, T. (2004). The visual salience of algebraic transformations. *Journal for Research in Mathematics Education*, 35(4), 224–257.
- Ladson-Billings, G. (1998). It doesn’t add up: African American students’ mathematics achievement. In C. Malloy & L. Brader-Araje (Eds.), *Challenges in mathematics education of African American children: Proceedings of the Benjamin Banneker Association Leadership Conference* (pp. 7–14). Reston, VA: National Council of Teachers of Mathematics.
- Lobato, J., & Ellis, A. B. (In Press). *Essential understandings project: Ratios, proportions, and proportional reasoning* (Gr. 6–8). National Council of Teachers of Mathematics.
- Lobato, J., & Thanheiser, E. (2002). Developing understanding of ratio and measure as a foundation for slope. In B. Litwiller & G. Bright (Eds.), *Making sense of fractions, ratios, and proportions: 2002 yearbook* (pp. 162–175). Reston, VA: National Council of Teachers of Mathematics.
- Moreno, R., & Mayer, R. E. (1999). Multimedia-supported metaphors for meaning making in mathematics. *Cognition and Instruction*, 17, 215–248.
- Moses, R. & Cobb, C. (2001). *Radical equations: Math literacy and civil rights*. Boston, MA: Beacon Press.
- National Commission of Excellence in Education. (1983). *A nation at risk: The imperative for educational reform*. Washington, DC: U.S. Department of Education.
- National Council of the Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics: A quest for coherence*. Reston, VA: Author.
- National Mathematics Advisory Panel. (2008). *The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.
- National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education*. Washington, DC: National Academies Press.
- National Research Council. (1998). *The nature and role of algebra in the K–14 curriculum*. Washington, DC: National Academies Press.
- National Research Council. (2000). *Mathematics education in the middle grades: Teaching to meet the needs of middle grades learners and to maintain high expectations*. Washington, DC: National Academies Press.
- Prawat, R. (1991). The value of ideas: The immersion approach to the development of thinking. *Educational Researcher*, 20(2), 3–10.
- RAND Mathematics Study Panel. (2003). *Mathematical proficiency for all students: Toward a strategic research and development program in mathematics education*. Santa Monica, CA: RAND.
- Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other? *Journal of Educational Psychology*, 91, 175–198.
- Schoenfeld, A. (1995). Is thinking about “algebra” a misdirection? In C. Lacampagne, W. Blair, & J. Kaput (Eds.), *The Algebra Colloquium. Volume 2: Working Group Papers* (pp. 83–86). Washington, DC: U.S. Department of Education.
- Siegler, R. S. (2003). Implications of cognitive science research for mathematics education. In J. Kilpatrick, W. B. Martin, & D. Schifter, (Eds.), *A research companion to principles and standards for school mathematics* (pp. 219–233). Reston, VA: National Council of Teachers of Mathematics.
- Smith, E. (2002). Stasis and change: Integrating patterns, functions, and algebra. In J. Kilpatrick, W. G. Martin, & D. Schifter, (Eds.), *A research companion to principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- Smith, J., diSessa, A., & Roschelle, J. (1993). Misconceptions reconceived: A constructivist analysis of knowledge in transition. *Journal of the Learning Sciences*, 3(2), 115–163.
- Stacey, K., & MacGregor, M. (1997). Building foundations for algebra. *Mathematics Teaching in the Middle School*, 2(4), 253–260.
- Wearne, D., & Kouba, V. L. (2000). Rational numbers. In E.A. Silver & P. A. Kenney (Eds.), *Results from the seventh mathematics assessment of the National assessment of Educational Progress* (pp. 163–191). Reston, VA: National Council of Teachers of Mathematics.
- Wenger, R. H. (1987). Cognitive science and algebra learning. In A.H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 217–251). Hillsdale, NJ: Erlbaum.

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